MATHEMATICAL SIMULATION OF IMPEDANCE DIFFRACTION GRATINGS

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ABSTRACT

In this paper the eigenwaves of periodic impedance diffraction grating (DG), reflection and transmittance on finite DG are under investigation. The approximate boundary conditions (ABC) were used for simulations. The refined expression for impedance dielectric strips is obtained. Singular part of approximate integral equations (IE) is extracted and analytically transformed. As the result the IE of second type with smooth kernel has been carried out and solved with collocation method (CM) and Galerkin method (GM).

INTRODUCTION

The application of impedance boundary conditions (IBC) greatly simplifies the solution of boundary problem for structure consisted of fine dielectric layers. It became a reason of huge amount of work appearance dedicated as to IBC obtaining, as to their application for concrete electrodynamics problems solution. For example, the problem of wave reflection from dielectric grating was solved in paper [1] with IBC method. The main disadvantage of IBC for dielectric structure is narrow frameworks of application. In theory these conditions are correct if a layer thickness is rather smaller than a wavelength. In practice these conditions give satisfactory accuracy if a layer thickness is smaller than a wavelength. IBC application area may be enlarged, for example, due to modernization described in [2]. In present paper we elaborate one more type of IBC. The IE described here are obtained by means of approximate solution of a rigorous IE [3]. Our IE become more simple in the case of $\bar{E}(0,0,E)$ wave diffraction on two-dimensional structure (Fig. 1)

$$E(x,y)=E^e(x,y)+k^2\int_S \tau(x',y')E(x',y')g(x,x',y,y')ds', x,y\in S \tag{1}$$

where $E^e(x,0)$ is the external field, $\tau=\varepsilon-\varepsilon_e$, k - wave number, $\varepsilon,\varepsilon_e$ - dielectric permittivity of local inhomogeneity and surrounding environment, S - cross- section of the local inhomogeneity, $g(x,x',y,y')$ - Green function (GF), in our case the GF is one f planar DW. The GF for planar DW consisted of an arbitrary number of layers is obtained. Half-analytical solution for this IE is already described in [4]. Let's consider the solution (1) for a single inhomogeneity. Process to large quantity of inhomogeneities is evident. Let's find an approximate IE solution for rectangular inhomogeneity.

Considering $E(x',y')\approx E(x')\exp\left(-ik\sqrt{\varepsilon}\,y'\right)$, where $E(x')$ is unknown function, we have

$$\int_{-a}^{a}E(x',y')g(x,x',y,y')dy'\approx g(x,x'y,y)\int_{-a}^{a}E(x',y')dy'\approx g(x,x'y,y)2\frac{\sin\left(k\sqrt{\varepsilon}\,a\right)}{k\sqrt{\varepsilon}}$$

And as the result we arrive at following one-dimensional IE.

$$\int_{-l}^{l}E(x')G(x,x')dx'-E(x)/k^2\delta=-E^c(x,0)/k^2\delta\ ,$$

where $G(x,x')=g(x,x',0,0)$,

$$\delta=2\frac{\sin\left(k\sqrt{\varepsilon}\,a\right)}{k\sqrt{\varepsilon}}\tau\,. \tag{2}$$

At $k\sqrt{\varepsilon}\,a<<1$ the expression for δ is well-known

$$\delta=2a\left(\varepsilon-\varepsilon_c\right). \tag{3}$$

It gives us an ability to write the IE as follow

$$\int_{-l}^{l}J(x')G(x,x')dx'-J(x)/k^2\delta=-E^c(x,0)\,, \tag{4}$$

where $J(x)=k^2\delta\,E(x)$. We would obtain the same IE if use approximate IBC from [4] with supposition $J(x)=-i\omega\,\mu_0\left[H(x,a)-H(x,-a)\right]$. Considering $\delta=\infty$ in (4) yields the IE for inhomogeneity as metal strip. The current $J(x)$ has a singularity on the border of metal strip. One of the methods to avoid this singularity is to change variables as - $x=l\cos\varphi$, $f(\varphi)=Jl\sin\varphi$. Numerical experiments have shown that the changing of variables gives good convergence for impedance strips too, because the current on these strips increases at the border as well. Our IE (4) in this case may be written as

$$\int_{0}^{\pi}f(\varphi')g(\varphi,\varphi')d\varphi'-f(\varphi)/k^2\delta\,l\sin\varphi=-E^c(l\cos\varphi,0)\,, \tag{5}$$

where $g(\varphi,\varphi')=G(l\cos\varphi,l\cos\varphi')$. Note, that the condition $\varphi'\to\varphi$ yields $g(\varphi,\varphi')\approx g_0(\varphi,\varphi')=-\log\left|2(\cos\varphi-\cos\varphi')\right|/2\pi$. Then we extract singular part of kernel IE (5) and transform this IE as follow

$$\int_{0}^{\pi}\left[f(\varphi')g(\varphi,\varphi')-f(\varphi)g_0(\varphi,\varphi')\right]d\varphi'+f(\varphi)\int_{0}^{\pi}g_0(\varphi,\varphi')d\varphi'-f(\varphi)/k^2\delta\ l\sin\varphi=-E^c(l\cos\varphi,0) \tag{6}$$

The first integral in (6) has no singularity, the second one is equal to zero. Simplified IE (6) we solved with collocation method. To calculate the integral the highest calculation accuracy formula (formula of quadrangles) has been used. The second way to solve (6) is the Galerkin method (GM) with $\cos j\varphi'$ functions as basical ones. Upon transfer to the x coordinate they correspond to the first kind of Chebyshev polynomials. There is a

sense to use $\cos m\varphi$ only for metal strips, for impedance ones the function $\sin m\varphi$ may be applied.

RESULTS AND CONCLUSIONS

The methods elaborated have a good convergence. The number of collocation points and number of basical functions per one strip is from 3 up to 7. Therefore, the wave diffraction by high number of strips (up to 100) it is possible to investigate. The CM codes calculate ten time faster then GM ones.

Therefore, the eigenwaves of periodic impedance diffraction grating are investigated. Dispersion characteristics, the windows of transparency and phase synchronism conditions for first and second harmonics (Fig 1, 2) have been obtained as the result of the investigation. Table illustrates the comparison of waves propagation coefficients (for DG placed over planar waveguide h=1; d/h=0,5; ε =4,0; ε_1=4,0; ε_2=2,1; ε_3=1,0; a/h=0,2; l/h=0,25; N=1 is a number of strips) we obtained by rigorous method (variant 1) and by approximate one described above (variants 2 and 3). Clearly seen, that the impedance we introduced gives higher accuracy for thick strips ($k\sqrt{\varepsilon}a \le 1$). Analogous results are obtained for reflection coefficient $|S_{11}|$ from dielectric inhomogeneity [3].

Variant	λ		
	3,5	5,0	6,5
1	1,7328	1,6827	1,6064
2	1,7247	1,6996	1,6198
3	1,6873	1,7284	1,6349

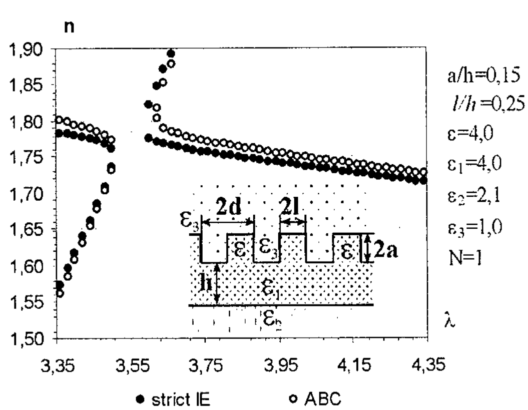

Fig.1

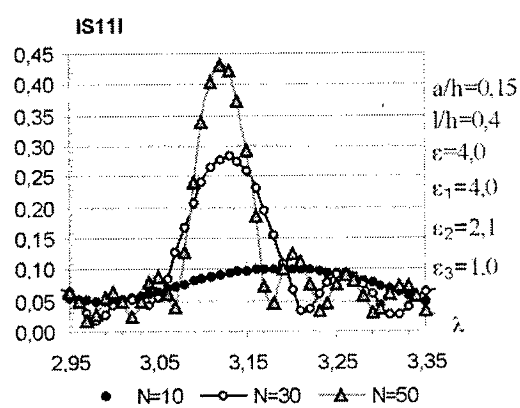

Fig. 2

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